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NOTE

A procedure for denoising dual-axis swallowing accelerometry signals

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Abstract
Dual-axis swallowing accelerometry is an emerging tool for the assessment of dysphagia (swallowing difficulties). These signals however can be very noisy as a result of physiological and motion artifacts. In this note, we propose a novel scheme for denoising those signals, i.e. a computationally efficient search for the optimal denoising threshold within a reduced wavelet subspace. To determine a viable subspace, the algorithm relies on the minimum value of the estimated upper bound for the reconstruction error. A numerical analysis of the proposed scheme using synthetic test signals demonstrated that the proposed scheme is computationally more efficient than minimum noiseless description length (MNDL)-based denoising. It also yields smaller reconstruction errors than MNDL, SURE and Donoho denoising methods. When applied to dual-axis swallowing accelerometry signals, the proposed scheme exhibits improved performance for dry, wet and wet chin tuck swallows. These results are important for the further development of medical devices based on dual-axis swallowing accelerometry signals.

Keywords: dual-axis swallowing accelerometry signals, denoising, wavelet transformation

1. Introduction

Swallowing accelerometry is a potentially informative adjunct to bedside screening for dysphagia (Reddy *et al* 2000, Das *et al* 2001, Chau *et al* 2005, Lee *et al* 2006). These measurements are minimally invasive, requiring only the superficial attachment of a sensor anterior to the thyroid notch. Even though single-axis accelerometers were traditionally used for swallowing accelerometry (Reddy *et al* 2000, Das *et al* 2001, Chau *et al* 2005, Lee *et al* 2006), recent studies have shown that dual-axis accelerometers can capture more of the clinically relevant information (Lee *et al* 2008, Sejdić *et al* 2009). Nevertheless, such measurements are inherently very noisy due to various physiological and motion artifacts (Lee *et al* 2008, Sejdić *et al* 2009). Denoising of dual-axis swallowing accelerometry signals is therefore essential for the development of a robust medical device based on these signals.

Estimation of unknown signals in white Gaussian noise has been dealt with in many recent contributions (e.g., Donoho and Johnstone 1994, Beheshti and Dahleh 2005, Jansen 2001) and wavelet denoising has been proposed as a valuable option (Donoho and Johnstone 1994). Wavelet denoising removes the additive white Gaussian noise from a signal by zeroing the wavelet coefficients with small absolute value. The suggested optimal threshold is equal to $\sigma_\varepsilon\sqrt{2\log N}$ where σ_ε^2 is the variance of the additive noise and N is the length of the signal (Donoho and Johnstone 1994). This approach requires knowledge of the noise variance, which can be estimated from the wavelet coefficients at the finest scale (Donoho and Johnstone 1994, Mallat 1999). However, wavelet denoising with the suggested optimal threshold does not necessarily produce the optimal results for signals that are not smooth, i.e. signals with noiseless coefficients being of very small amplitude for a large number of basis functions (Beheshti and Dahleh 2005). Recent attempts to overcome this shortcoming have yielded methods that can suffer from high computational complexities for very long signals such as those considered in this note, and do not necessarily reach the optimal results (Beheshti and Dahleh 2005).

The main contribution of this note is a computationally efficient algorithm for the denoising of long duration dual-axis swallowing accelerometry signals. The algorithm achieves low computational complexity by performing a search for the optimal threshold in a reduced wavelet subspace. To find this reduced subspace, the proposed scheme uses the minimum value of the estimated reconstruction error. By finding this value, the proposed approach also achieves a smaller reconstruction error than MNDL-, SURE-based and Donoho's approaches. This finding has been confirmed for both synthetic test signals and dual-axis swallowing accelerometry signals.

The note is organized as follows: in section 2, denoising is considered in detail and a novel algorithm for denoising swallowing accelerometry signals is described. Section 3 presents the results of the numerical analysis conducted using both synthetic test signals and actual dual-axis accelerometric recordings. The proposed method is compared against three other established wavelet denoising algorithms.

2. Proposed approach

Consider N noisy discrete-time observations

$$x(n) = f(n) + \varepsilon(n) \tag{1}$$

where $n = 0, \ldots, N-1$, $f(n)$ is a sampled version of a noiseless continuous signal and $\varepsilon(n)$ is the additive white Gaussian noise drawn from $\mathcal{N}\left(0, \sigma_\varepsilon^2\right)$. Assume that $f(n)$ can be expanded using basis functions, $b_k(n)$, on the observation space, $\mathcal{B}_N$:

$$f(n) = \sum_{k=1}^{N} c_k b_k(n) \tag{2}$$

where

$$c_k = \langle b_k(n), f(n)\rangle \tag{3}$$

and $\langle p, q\rangle$ denotes the inner product of vectors p and q. However, given the noisy observations, the coefficients, c_k, can be only approximated as follows:

$$\widehat{c}_k = \langle b_k(n), x(n)\rangle = c_k + \langle b_k(n), \varepsilon(n)\rangle. \tag{4}$$

2.1. Denoising and reconstruction error

If $f(n)$ can be described with M nonzero coefficients, where $M \leqslant N$, then many estimated coefficients, $\widehat{c}_k$, represent samples of a zero mean Gaussian random variable with variance σ_ε^2. A classical approach known as wavelet denoising diminishes the effects of noise (Donoho and Johnstone 1994) by first expanding the noisy signal in terms of orthonormal bases of compactly supported wavelets. The estimated coefficients below some threshold, τ, are disregarded either by hard or soft thresholding. The value of τ is always chosen based on an attempt to minimize the so-called reconstruction error, r_e:

$$r_e = \frac{1}{N}\|f(n) - \widehat{f}(n)\|^2 \tag{5}$$

where $\|\cdot\|$ denotes the Euclidean norm and $\widehat{f}(n)$ represents the estimated noiseless signal. When the hard thresholding procedure is implemented, r_e is a sample of random variable R_e that has the following expected value (Beheshti and Dahleh 2005):

$$E\{R_e\} = \frac{m}{N}\sigma^2 + \frac{1}{N}\|\Delta_m\|^2 \tag{6}$$

where m represents the number of coefficients describing $f(n)$ in some subspace of $\mathcal{B}_N$ and Δ_m is a vector of length $N - m$, representing the coefficients of bases that are not selected to describe the unknown signal. In reality, r_e is not available and only the number of coefficients not disregarded by the thresholding operation, $\widehat{m}$, is known. In a recent contribution, probabilistic upper and lower bounds for r_e were derived based on the available data error (Beheshti and Dahleh 2005):

$$d_e = \frac{1}{N}\|x(n) - \widehat{f}(n)\|^2. \tag{7}$$

Therefore, it has been shown that for the hard thresholding case the upper bound for r_e is equal to

$$\begin{aligned} r_{eub}(\widehat{m}(\tau), \sigma^2, \alpha, \beta) &= \frac{\sigma_\varepsilon^2\sqrt{2\widehat{m}(\tau)}}{N}\left(\sqrt{2\widehat{m}(\tau)} + \beta\right) + d_e - \sigma_\varepsilon^2 \\ &\quad + \frac{2\alpha\sigma_\varepsilon}{\sqrt{N}}\sqrt{\frac{\alpha^2\sigma_\varepsilon^2}{N} + d_e - \left(1 - \frac{\widehat{m}(\tau)}{N}\right)\frac{\sigma_\varepsilon^2}{2} + \frac{2\alpha^2\sigma_\varepsilon^2}{N}} \end{aligned} \tag{8}$$

where α and β represent the parameters for validation probability ($p_v = Q(\alpha)$) and confidence probability ($p_c = Q(\beta)$), with $Q(\cdot)$ for an argument λ being defined as $Q(\lambda) = \int_{-\lambda}^{+\lambda}(1/\sqrt{2\pi})\,\mathrm{e}^{-x^2/2}\,\mathrm{d}x$. In addition, $\widehat{m}(\tau)$ denotes the number of bases whose expansion coefficients are greater than τ in some subspace of $\mathcal{B}_N$. For complete derivations and information on the lower and upper bounds please refer to Beheshti and Dahleh (2005).

It should be noted that for some values of $\widehat{m}$ the reconstruction error given by equation (5) and its upper bound given by equation (8) achieves a minimum due to the bias-to-variance tradeoff (Beheshti and Dahleh 2005, Jansen 2001). Beheshti *et al* borrowed the principle of MNDL from coding theory to find such a minimum value (Beheshti and Dahleh 2005). Also, it has been demonstrated that smaller reconstruction errors can be achieved with MNDL-derived thresholds than with the threshold defined in Donoho and Johnstone (1994).

2.2. *Algorithm for determining optimal threshold*

The MNDL-based approach can be computationally expensive for very long data sets since the bases are incrementally added to the subspace describing the unknown signal (Beheshti and Dahleh 2005). Considering the length of acquired dual-axis accelerometry signals ($\gg 10^5$ points), an attempt should be made to minimize the search space, while choosing a threshold that minimizes the reconstruction error. In addition, the numerical experiments conducted in Beheshti and Dahleh (2005) showed that in some cases the MNDL-based approach can yield higher reconstruction errors than Donoho's approach.

In light of the computational and reconstruction limitations of the MNDL-based approach, a new denoising strategy is proposed here. The goal of this new approach is twofold. First, it should be computationally efficient. Second, it should attain a minimum reconstruction error. Minimization of the search space can be achieved by exploiting the fact that the optimal threshold given in Donoho and Johnstone (1994) is usually larger than the actual threshold which minimizes the reconstruction error (Beheshti and Dahleh 2005, Jansen 2001). The algorithm for determining the optimal threshold is defined through the following steps.

(i) Estimate the variance of the noise ε from the median, MED_x, of $N/2$ wavelet coefficients at the finest scale (Donoho and Johnstone 1994, Mallat 1999):

$$\widehat{\sigma_\varepsilon} = \frac{\mathrm{MED}_x}{0.6745}. \tag{9}$$

(ii) Based on the estimated noise variance and for each τ selected from a set $0 < \tau \leqslant \widehat{\sigma_\varepsilon}\sqrt{2\log(N)}$, evaluate the upper bound given by (8). Use the soft thresholding procedure to compute the data error required for the evaluation of the upper bound.

(iii) Determine the optimal threshold for wavelet denoising as

$$\tau_{\mathrm{opt}} = \mathrm{argmin}_\tau r_{eub}\left(\widehat{m}(\tau), \widehat{\sigma_\varepsilon}^2, \alpha, \beta\right). \tag{10}$$

(iv) Denoise a recording using the optimal value of threshold, τ_{opt}, and the soft thresholding procedure.

The above procedure is repeated independently for signals acquired from each axis of a dual-axis accelerometer. Unlike the MNDL-based approach, soft thresholding is applied in the above steps, since it yields an estimated signal as smooth as the original signal with high probability (Donoho 1995). Hard thresholding can produce abrupt artificats in the recovered signal (Chang *et al* 2000), leading to a higher reconstruction error.

It should be pointed out that the proposed method is an ad hoc method based on the MNDL approach. The used upper bound for the reconstruction error (equation (8)) assumes the hard thresholding approach. However, we implemented the soft thresholding for the proposed method. Therefore, the proposed method cannot guarantee to provide the (global) minimum of the reconstruction error.

3. Numerical analysis

The results of a two-step numerical analysis are presented in this section. First, the performance of the proposed algorithm is examined using two test signals. The goal of this analysis is to compare the performance of the proposed scheme against that of other well-established techniques under well-controlled conditions. In the second step, the proposed denoising algorithm is applied to the acquired dual-axis swallowing accelerometry signals. The goal is to understand the benefits of the proposed approach in the context of a real biomedical application.

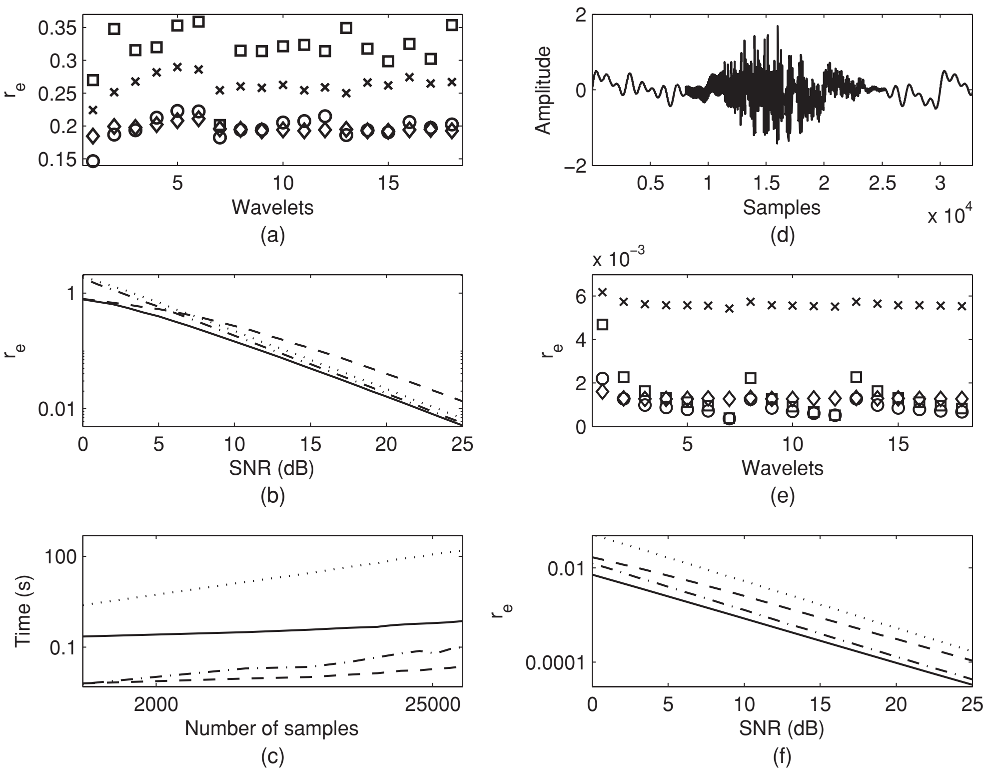

Figure 1. Numerical analysis for the two synthetic signals: reconstruction errors for 18 different wavelets are shown in (a) for the Block signal and in (e) for the sample swallow-like signal (□ = Donoho, ⋄ = SURE, × = MNDL, ○ = proposed). The reconstruction errors as a function of SNR are shown in (b) for the Block signal using Haar wavelet and in (f) for the synthetic swallowing signal using Meyer wavelet (dashed line = Donoho; dash-dotted line = SURE; dotted line = MNDL; solid line = proposed). Time required to execute denoising task for Block signals of increasing length is depicted in (c). The synthetic swallowing signal is depicted in (d).

3.1. Performance analysis using synthetic test signals

The first test signal is the so-called Blocks signal as defined in Donoho (1995), which is a standard signal used in the analysis of various denoising schemes (e.g. Beheshti and Dahleh 2005). Assuming that the length of the signal is $N = 1024$ points, the reconstruction error is evaluated for three methods: the proposed method, Donoho's approach (Donoho 1995), the MNDL-based method (Beheshti and Dahleh 2005) and a new SURE-based approach (Luisier *et al* 2007). The first test is to numerically examine which of the four schemes provides the lowest reconstruction error for 18 mother wavelets (Haar wavelet, Daubechies wavelets with the number of vanishing moments varying between two and six, Meyer wavelet, Coiflet wavelets with the order varying between one and five, and Symlet wavelets with the order varying between two and seven). The signal is contaminated with zero-mean additive white Gaussian noise, and the signal-to-noise ratio (SNR) is 10 dB. For each mother wavelet, 1000 realizations are used. $\alpha = 10$ and $\beta = 40$ are used for both the MNDL-based approach and the proposed method. The reconstruction errors for the proposed method (circles), the MNDL-based denoising (x's), the SURE-based approach (diamonds) and Donoho's approach (squares) are shown in figure 1(a). Amongst the 18 wavelet functions, considered, the Haar wavelet (the wavelet indexed as 1 on the x-axis of figure 1(a)) provides the smallest reconstruction error, since the structure of the wavelet closely resembles the structure of the signal.

The next task is to examine the reconstruction error under various SNR values with the Haar wavelet. One thousand realizations are used for each SNR value yielding the results depicted in figure 1(b). From the graph, it is clear that the proposed method (solid line) provides the smallest error for various SNR levels, with the MNDL-based (dotted line) and SURE-based (dash-dotted line) methods also providing a small error. Donoho's approach (dashed line) consistently yields the highest reconstruction error. Despite the small reconstruction error over different SNR levels, the MNDL-based approach suffers from high computational complexity. To further understand the computational bottlenecks, the SNR value is kept constant at 10 dB, but the length of the Blocks signal is varied between $N = 2^{10}$ and $N = 2^{15}$ points. The duration required to execute the specific algorithms is tracked using built-in MATLAB functions. The time to complete the denoising task, averaged over ten realizations of the Block signal at each signal length, is reported in figure 1(c). As expected, as N increases, there is an obvious upward trend for all four algorithms. Donoho's approach (dashed line) is the least computationally expensive. However, for the MNDL-based approach (dotted line) the time required to complete the task increases significantly with signal length. For example, the average duration required for the MNDL-based approach to denoise a signal with length of $N = 2^{15}$ points is 157 s. On the other hand, the time required by the proposed algorithm (solid line) to denoise the same signal is 0.74 s. In fact, computation time of the proposed method increases logarithmically with signal length (the duration is approximately equal to $\log_{10}(N^{0.35})$).

To more closely mimic a real swallowing scenario, the test signal shown in figure 1(d), is used in the analysis. The signal is defined as

$$f(n) = \begin{cases} f_o(n) + 0.6\cos(210\pi nT) & 8100 \leqslant n \leqslant 16\,430 \\ f_o(n) + 0.5\cos(140\pi nT) & 11\,400 \leqslant n \leqslant 18\,330 \\ f_o(n) + 0.2\cos(120\pi nT) & 13\,200 \leqslant n \leqslant 25\,230 \\ f_o(n) + 0.4\cos(160\pi nT) & 12\,250 \leqslant n \leqslant 23\,400 \\ f(n)w(n) & 8100 \leqslant n \leqslant 25\,230 \end{cases} \tag{11}$$

where $w(n)$ is the Gaussian window with standard deviation $\sigma_g = 1.9$ and

$$f_o(n) = 0.1\sin(8\pi nT) + 0.2\sin(2\pi nT) + 0.15\sin(20\pi nT) + 0.15\sin(6\pi nT) + 0.12\sin(14\pi nT) + 0.1\sin(4\pi nT) \tag{12}$$

with $0 \leqslant n \leqslant N-1$, $N = 35\,000$ and $T = 10^{-4}$ s. The duration of the signal is based on previously reported swallow durations (Sejdić *et al* 2009). It should be mentioned that this signal only mimics a realistic signal, and does not represent a model of a swallow. The same group of wavelets as in the Blocks signal analysis are used to examine the reconstruction error. It is assumed again that the signal is contaminated with additive zero-mean Gaussian noise and SNR = 10 dB. For this particular signal, the Meyer wavelet (indexed by number 7 in figure 1(e)) achieved the smallest reconstruction error since the structure of the wavelet resembles the structure of the signal. It should be pointed out that the MNDL-based method consistently provides the highest error for all considered wavelets. Given that the method is sensitive to the choice of α and β (Beheshti and Dahleh 2005), we varied the two parameters to further examine the obtained error. The MNDL method still maintained the highest reconstruction error for this particular signal. The main reason for these results is the hard thresholding procedure used in this method. Consequently, the better results are indeed expected with an approach implementing a soft thresholding procedure (e.g. Chang *et al* 2000). As the next step, the reconstruction error is evaluated using the Meyer wavelet

Table 1. Performance measure for the considered approaches.

		Dry	Wet	WCT	Overall
Donoho	A-P	5.8 ± 1.8	5.4 ± 1.3	2.9 ± 1.7	4.8 ± 2.1
	S-I	4.1 ± 2.7	3.4 ± 2.0	2.2 ± 1.9	3.3 ± 2.4
MNDL	A-P	5.4 ± 0.3	5.4 ± 0.3	5.2 ± 0.3	5.3 ± 0.3
	S-I	5.1 ± 0.5	5.1 ± 0.7	5.0 ± 0.4	5.0 ± 0.5
SURE	A-P	5.8 ± 4.2	6.9 ± 4.3	7.1 ± 3.9	6.6 ± 4.2
	S-I	6.5 ± 6.1	6.3 ± 3.9	5.1 ± 5.5	6.0 ± 5.4
Proposed	A-P	9.3 ± 1.8	9.0 ± 1.6	7.5 ± 1.2	8.6 ± 1.8
	S-I	7.8 ± 1.9	7.3 ± 1.6	7.0 ± 1.4	7.3 ± 1.7

for various SNR values for all four approaches. From the results shown in figure 1(f), it is obvious that the proposed method (solid line) achieves a significantly smaller reconstruction error than the other three methods.

3.2. Denoising dual-axis swallowing accelerometry signals

3.2.1. Experimental protocol. During a 3 month period, 408 participants (aged 18–65) were recruited at a public science center. All participants provided written consent. The study protocol was approved by the research ethics boards of the Toronto Rehabilitation Institute and Bloorview Kids Rehab, both located in Toronto, Ontario, Canada. A dual-axis accelerometer (ADXL322, Analog Devices) was attached to the participant's neck (anterior to the cricoid cartilage) using double-sided tape. The axes of acceleration were aligned to the anterior–posterior (A-P) and superior–inferior (S-I) directions. Data were band-pass filtered in hardware with a pass band of 0.1–3000 Hz and sampled at 10 kHz using a custom LabVIEW program running on a laptop computer.

With the accelerometer attached, each participant was cued to perform five saliva swallows (denoted as dry in table 1). After each swallow, there was a brief rest to allow for saliva production. Subsequently, the participant completed five water swallows (denoted as wet in table 1) by cup with their chin in the natural position (i.e. perpendicular to the floor) and five water swallows in the chin-tucked position (denoted as WCT in table 1). The entire data collection session lasted 15 min per participant.

3.2.2. Results of denoising. The acquired dual-axis swallowing accelerometry signals were denoised using the following schemes: Donoho's approach, the MNDL-based approach, the SURE-based approach and the proposed approach. In particular, a ten-level discrete wavelet transform using the Meyer wavelet with soft thresholding was implemented. Before denoising, the signals were pre-processed using inverse filters derived through system identification (Ljung 1999) to annul the effects of the data collection system on the acquired data. In order to compare the performance of the aforementioned denoising schemes, we used the following performance metric:

$$\Xi_{(\mathrm{dB})} = 10\log_{10}\left(\frac{E_f}{r_{eub}\left(\widehat{m}(\tau), \widehat{\sigma}_{\widehat{\varepsilon}}^2, \alpha, \beta\right)}\right) - 10\log_{10}\left(\frac{E_f}{E_{\widehat{\varepsilon}}}\right) \tag{13}$$

where E_f represents the approximate energy of the noise-free signal, and $E_{\widehat{\varepsilon}}$ represents an approximate variance of the white Gaussian noise. The approximate energy is calculated as $E_f = \widehat{\sigma}_x^2 - \widehat{\sigma}_{\widehat{\varepsilon}}^2$ (Sendur and Selesnick 2002, Chang *et al* 2000), where $\widehat{\sigma}_x^2$ is the variance of the observed signal, and $\widehat{\sigma}_{\varepsilon}^{2}$ represents the variance of the noise calculated by (9). Note that the first term in equation (13) represents the so-called signal-to-error ratio (Baykal *et al* 1997), while the second term represents an approximate SNR value of a noisy signal (Nowak 1999).

Using the performance metric given by (13), the results of the analysis are summarized in table 1. Donoho's approach provides the least amount of improvement, followed by the MNDL-based approach. The SURE-based approach achieves greater improvement in comparison to the other two aforementioned approaches. Nevertheless, as demonstrated by the results in table 1, the SURE approach exhibits strong variations in performance. The proposed approach provides the greatest performance improvement. On average, the greatest gain is over Donoho's approach (3.8 dB and 4.0 dB in the A-P and S-I directions, respectively), while smaller improvements were obtained over the SURE-based approach (2.0 dB and 1.3 dB in the A-P and S-I directions, respectively). Nevertheless, the proposed approach still provides a statistically significant improvement over the SURE-based approach in denoising the dual-axis swallowing accelerometry signals (Wilcoxon rank-sum test, $p \ll 10^{-10}$ for both directions). This improvement was achieved regardless of whether or not the different swallowing types were considered individually or as a group. As a last remark, it should be noted that these performance measure values were estimated using equation (13), which, from our experience with swallowing signals, provides a conservative approximation. In reality, we expect the gains in dB to be even greater.

4. Conclusion

In this note, a denoising algorithm was proposed for dual-axis swallowing accelerometry signals, which have potential utility in the non-invasive diagnosis of swallowing difficulties. This algorithm searches for the optimal threshold value in order to achieve the minimum reconstruction error for a signal. To avoid the high computational complexity associated with competing algorithms, the proposed scheme conducts the threshold search in a reduced wavelet subspace. Numerical analysis showed that the algorithm achieves a smaller reconstruction error than Donoho, MNDL- and SURE-based approaches. Furthermore, the computational complexity of the proposed algorithm increases logarithmically with signal length. The application of the proposed algorithm to dual-axis swallowing accelerometry signals demonstrated a statistically significant enhanced performance over the other three considered methods.

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