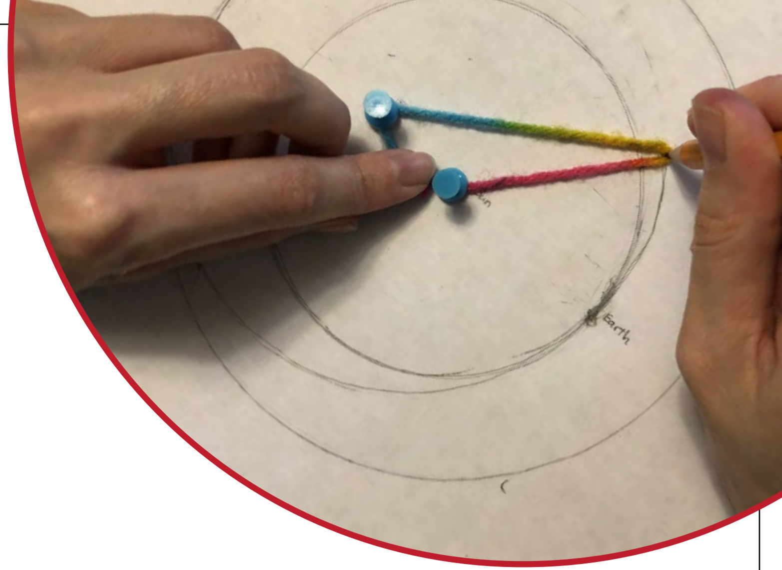
MARS MATH



Learn about the impact of orbits on space travel

What you'll need:

- Paper
- Corrugated cardboard (e.g. a pizza box)
- Tape or glue
- 2 push pins
- A pencil
- A ruler
- String
- Scissors
- A calculator (optional)

Activity setup:

1. Place the paper onto the cardboard and fasten in place with tape or glue.

2. Insert one push pin into the centre of the paper. This pin represents the position of the Sun.

3. Using a ruler, measure a point 5 cm to the right of the pin and draw a large dot with a pencil. This dot represents the Earth.

4. Measure and cut a 10-cm length of string. Tie the ends together.

5. Loop the string around the push pin and the tip of your pencil.

6. With the string as your guide and the pin as your anchor, draw a circle. This circle represents the orbit of the Earth around the Sun.

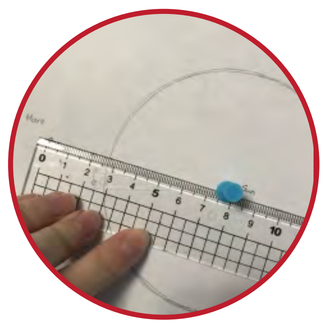
7. Use your ruler to measure a point 7.6 cm to the left of the pin and draw another dot. This dot represents Mars.

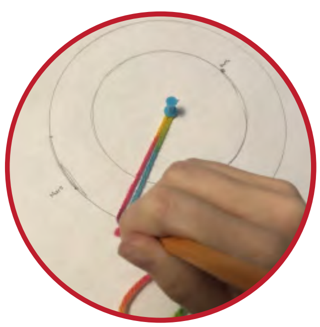
8. Cut a 15-cm length of string and tie the ends together. Repeat steps 5-6 to create another circle. This circle represents the orbit of Mars around the Sun.

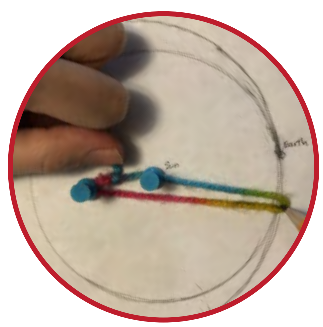
9. Measure a point 2.6 cm to the left of the first pin and insert the second push pin.

10. Place the 15-cm loop of string around the two pins and insert a pencil into the loop.

11. Stretch the string with the pencil to form a triangle, and then draw a curve between the Earth and Mars.

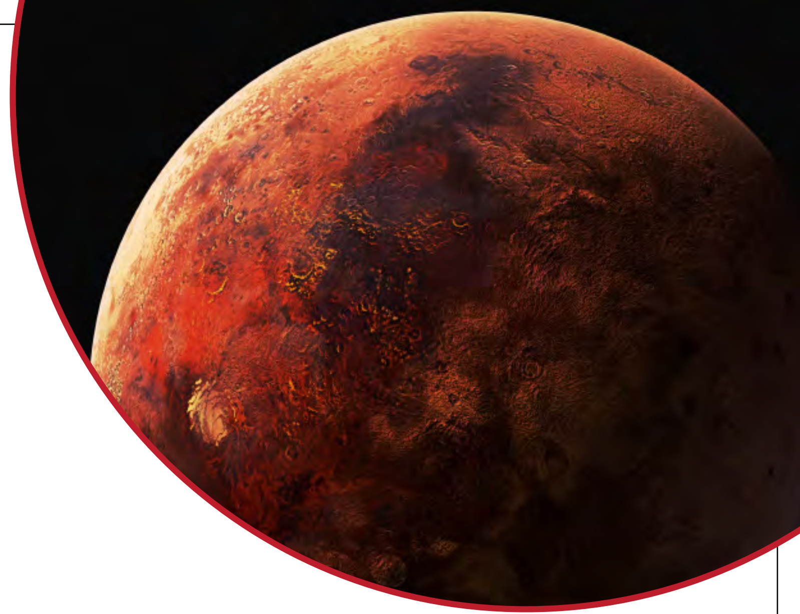






ONTARIO SCIENCE CENTRE
An agency of the Government of Ontario

MARS MATH



Learn about the impact of orbits on space travel

What's happening?

You have just learned how to draw two curved geometric shapes: a **circle** and an **ellipse**, which is like an oval or a "squashed" circle. Studying these shapes is key to understanding how objects move through space.

The curve you drew in the last step above is an ellipse representing one of the most efficient paths from Earth to Mars. Each pin is placed at a special point called a **focus** of the ellipse. The distance from one focus to your pencil to the other focus is constant due to the fixed length of your string. To test this, pick any point on the curve, measure the distance from each pin to that point and add those numbers together. Then, select a different point on your curve and repeat your measurements. Are the added numbers (the sum of the distances) the same?

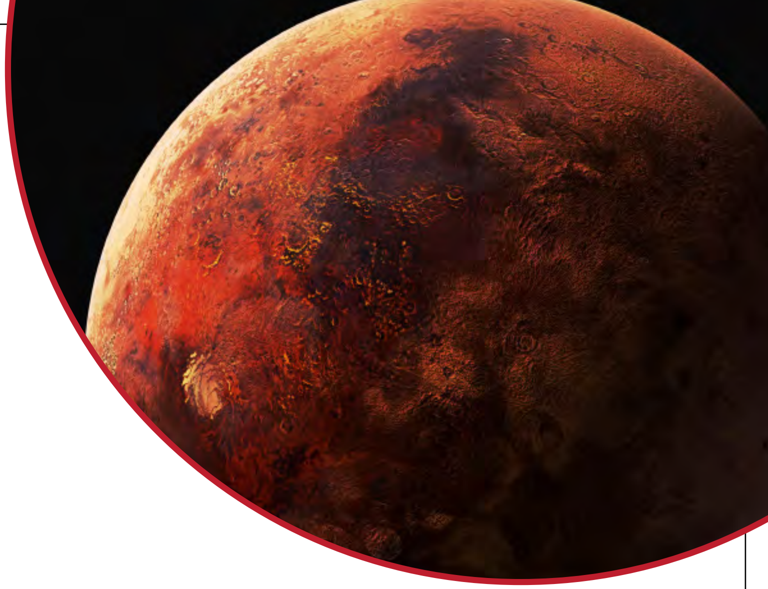
The circles you drew to represent the orbits of Earth and Mars around the Sun are also a type of ellipse. Each circle has one focus, which is represented in this activity by the pin placed at the centre.

How does it work?

An **orbit** is a path that an object takes around another object in space. Objects in orbit around the Earth can be natural, like our Moon, or human-made, like the International Space Station. Did you know that you are in orbit right now? The Earth and all the planets in our solar system are always in orbit around the Sun.

In this activity, you drew the orbits of Earth and Mars around the Sun as circles. Though planetary orbits are usually represented by circles, they are actually slightly elliptical—they have more than one focus.

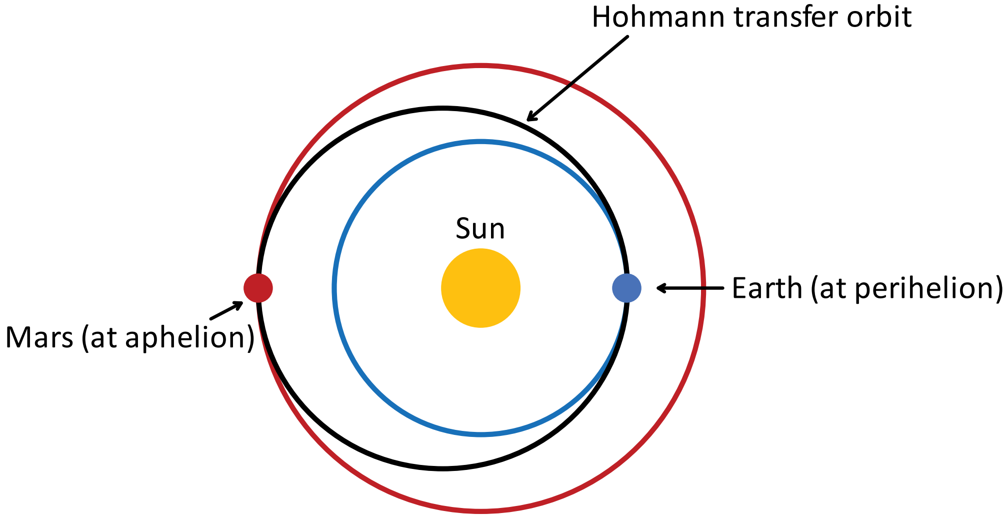
The curve you drew represents the most efficient path from Earth to Mars, known as the **Hohmann transfer orbit**. Though there are many available paths, this one requires the least energy. To arrive on Mars safely via the Hohmann transfer orbit, a spacecraft needs to be launched during a precise **launch window**. If it is launched too early or too late, it could miss the planet entirely, arriving when Mars is at a different place in its orbit.



MARS MATH

Learn about the impact of orbits on space travel

How does it work? (continued)



This launch window must be calculated precisely by taking into account the relative positions of the Earth and Mars in their respective orbits. At the time of launch, Earth needs to be at the point in the Hohmann transfer orbit that is closest to the Sun (**perihelion**). At the time of arrival, Mars should be at the point in the Hohmann orbit that is farthest from the Sun (**aphelion**).

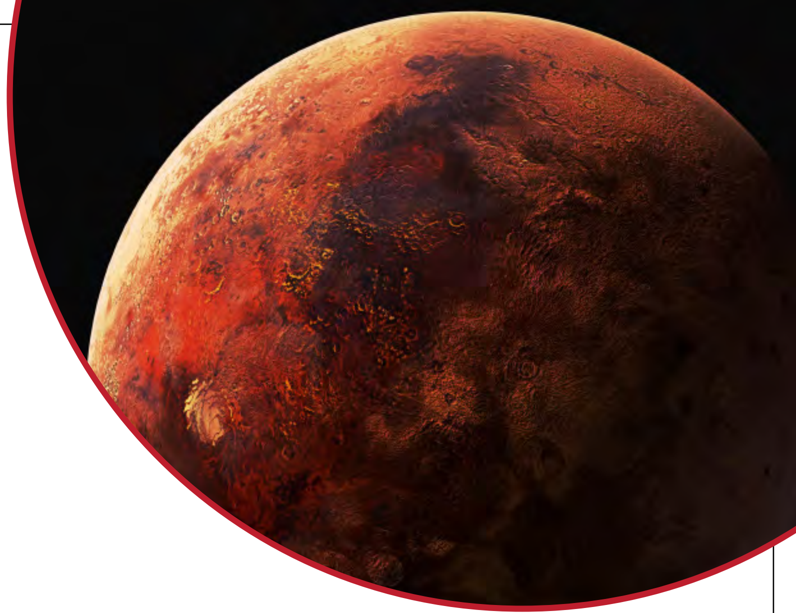
In July 2020, the launch window for reaching Mars via the Hohmann transfer orbit opened. At this time, the United States, China and the United Arab Emirates each launched missions to Mars with the Perseverance rover, the Tianwen-1 spacecraft and the Hope orbiter, respectively.

Did you know?

NASA used to rely on "human computers," many of whom were women, to calculate launch windows. One such computer was mathematician Katherine Johnson, one of the first African-American women to work as a NASA scientist. Her calculations of rocket trajectories and launch windows were integral to Project Mercury, NASA's first human spaceflight program, and the Apollo program.

MARS MATH

Calculate the next launch window



Once we've drawn the most energy efficient orbit, we can calculate the next launch window to Mars using this scale:

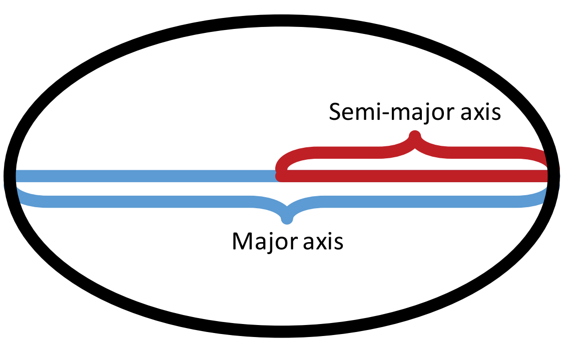
1 astronomical unit (AU or Earth–Sun distance) = 5 cm

Step 1
First, we use the scale to determine:

i. The distance from Mars to the Sun in AU:
7.6 cm ÷ 5 cm/AU = **1.52 AU**

ii. The length of the semi-major axis (half of the major axis) of the Hohmann transfer orbit in AU:

12.6 cm ÷ 5 cm/AU = 2.52 AU (major axis)
2.52 AU ÷ 2 = **1.26 AU** (semi-major axis)



Step 2
Next, we use Kepler's Third Law of Motion to determine the orbital period of the Hohmann transfer orbit. Kepler's Third Law of Planetary Motion states that:

$P^2 = a^3$

where P is the orbital period measured in years and a is the semi-major axis in AU.

If the semi-major axis is 1.26 AU, as determined in Step 1, then:

$P^2 = 1.26^3$
$P^2 = 2.000376$
$P = \sqrt{2.000376}$
$P =$ **1.4143464 years**

The orbital period of the Hohmann transfer orbit is approximately 1.41 years. To convert this into days, we multiply by 365.25:

1.41 x 365.25 = **515 days**

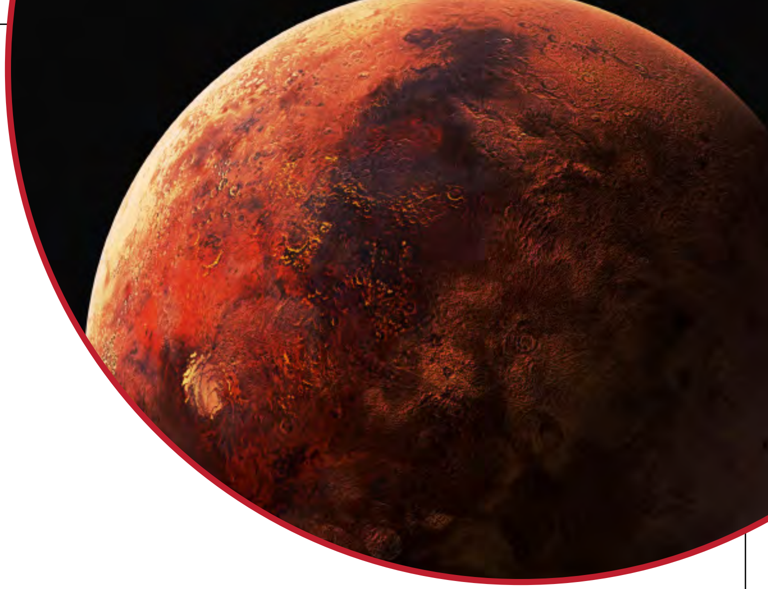
Therefore, using Kepler's Third Law, the orbital period of the Hohmann transfer orbit is 1.41 years or approximately 515 days.

Step 3
The spacecraft will travel half of the Hohmann transfer orbit to reach Mars. Its travel time to Mars will therefore be equivalent to half an orbit:

515 days ÷ 2 = **257.5 days**

It will take the spacecraft approximately 258 days to travel to Mars.

MARS MATH



Calculate the next launch window

Step 4
Next, we determine how far Mars will have rotated around the Sun in degrees (°) in the time it takes the spacecraft to travel from Earth to Mars. To calculate this, we must know that:

> Mars completes one revolution (360°) every 687 days

This means Mars moves approximately 0.524° (360° ÷ 687 days) everday.

In Step 3, we determined that the spacecraft will take about 258 days to travel to Mars:

> 258 days x 0.524° per day = **135.2° travelled**

Mars will travel approximately 135° of its orbit during the spacecraft's journey to Mars

Step 5
The spacecraft must be launched when Mars is in a specific position to ensure both meet at the Hohmann aphelion (the point farthest from the Sun). Using information from earlier steps, we can calculate this position.

We know that the spacecraft will travel 180° around the Hohmann transfer orbit before reaching Mars. During that time, Mars will have travelled approximately 135° in its own orbit

> 180° - 135° = **45°**

The next launch opportunity will occur when Mars is 45° ahead of the Earth in its orbit.

According to NASA's Table of Heliocentric Planetary Longitudes site, the next date that corresponds to this position will be in August 2022. This is when the next launch window for a trip to Mars via the Hohmann transfer orbit will occur.

References:

Activity based on NASA JPL Education Program "Let's Go to Mars"

Table of Heliocentric Planetary Longitudes